Lecture No. 37

Measure and Integration

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$$F(x) = \int_a^x f(t)\,d\lambda(t) \quad , a \leq x \leq b$$

is differentiable, with

$$F'(x) = f(x) \text{ a.e.}$$

<u>Step 1</u> We may assume $f \geq 0$.

$$f \in L_1[a,b],\ f = f^+ - f^-$$

$$f^+, f^- \geq 0;\ f^+, f^- \in L_1[a,b]$$

$$F(x) = \int_a^x f^+(t)\,dt - \int_a^b f^-(t)\,dt$$

$$= F_1(x) - F_2(x).$$

$$F_1(x) = \int_a^x f^+(t)\,dt$$

$$F_2(x) = \int_a^x f^-(t)\,dt$$

$$\Rightarrow F_1'(x) = f^+(x), \quad F_2'(x) = f^-(x)$$

$$(F_1 - F_2)'(x) = F_1'(x) \cancel{+F_1'} - F_2'(x)$$

$$= f^+(x) - f^-(x) \text{ a.e}$$

$$\Rightarrow \quad = f(x) \text{ a.e}$$

$$F'(x) = f(x) \text{ a.e. } (x)$$

$$F(x) = \int_a^x f(t)\, d\lambda(t)$$

F(x) is m. i

$f \geq 0$

$\Rightarrow \quad y > x$

$$F(y) - F(x) = \int_x^y f(t)\, d\lambda(t) \geq 0$$

II

$F : [a, b] \longrightarrow \mathbb{R}$

ab. cont

$\Longrightarrow$ $F' \in L_1[a, b]$

and $\int_a^x F'(t) \, d\lambda(t) = F(x)$

μ is counting measure

λ is Lebesgue measure

<u>Claim</u>

$$\underline{\lambda \ll \mu}.$$

Let $E \in \mathcal{L}_{\mathbb{R}}$, $\mu(E) = 0$

$$\Rightarrow E = \emptyset$$

$$\Rightarrow \lambda(\emptyset) = \lambda(E) = 0$$

$f \in L_1[a,b]$

$$\Longrightarrow F(x) = \int_a^x f(t)\, d\lambda(t), \quad x \in [a,b]$$

$F'(x)$ exists a. e. and

$F'(x) = f(x)$ a. e